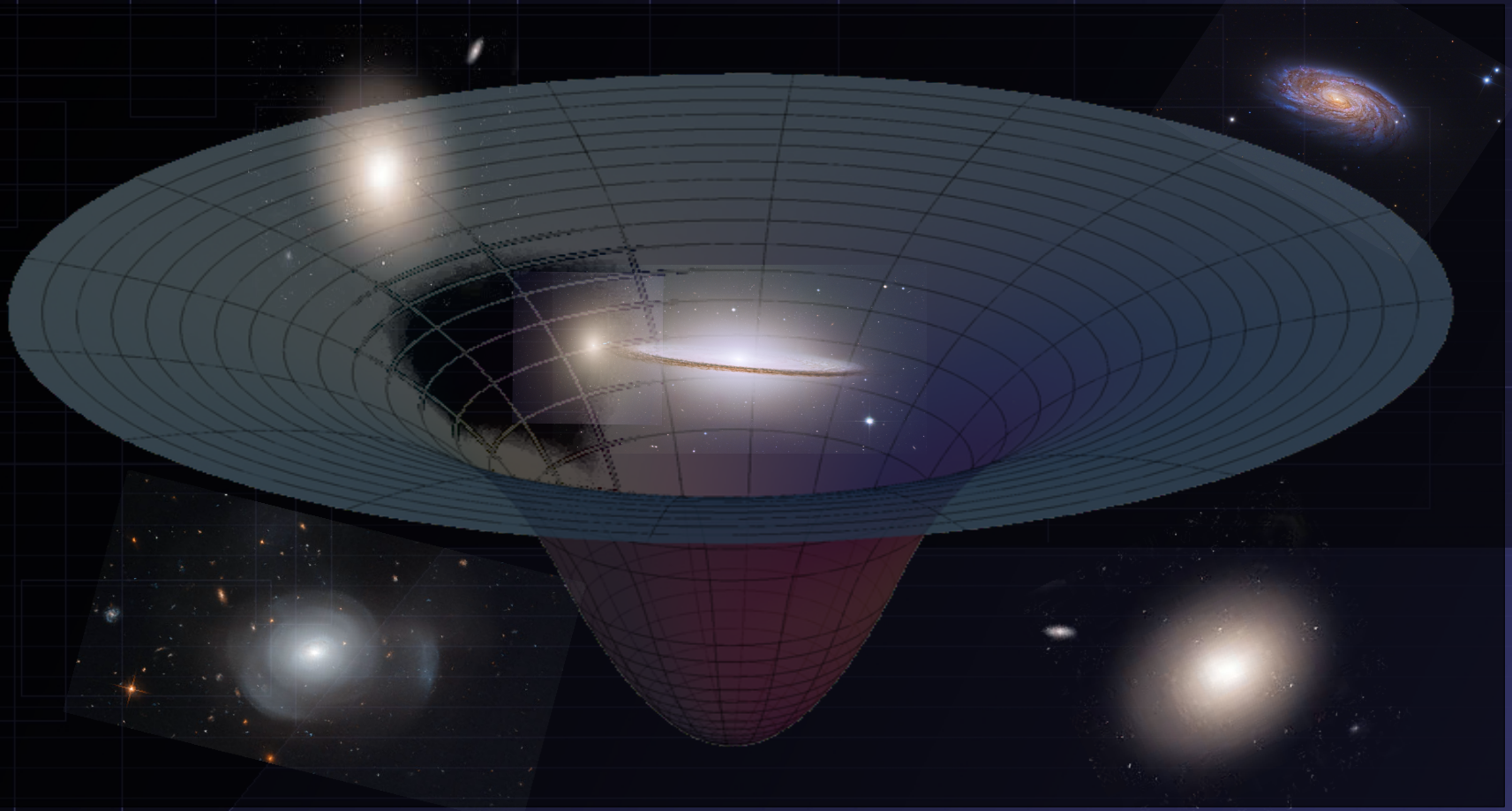


Frontier Cosmology with Galaxy Clusters

Christopher J. Miller
University of Michigan



What Roles Do Galaxy Clusters Play in Cosmology

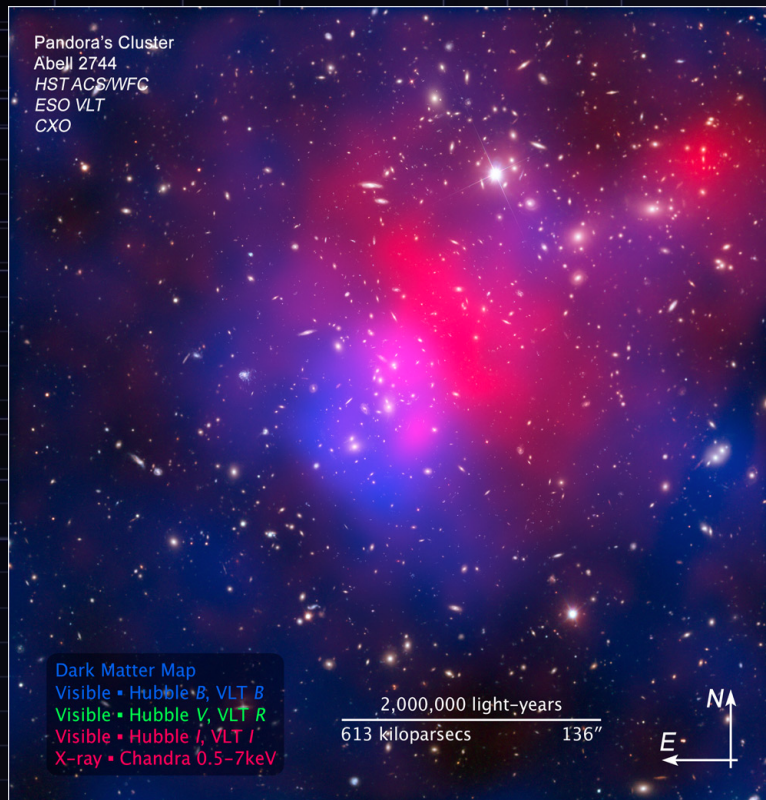
- Counting Statistics

- Simple:
Mass functions
Hi-z, high mass
- Spatial:
high mass peaks
2pt and/or $P(k)$

- Seeding the Growth of Structure

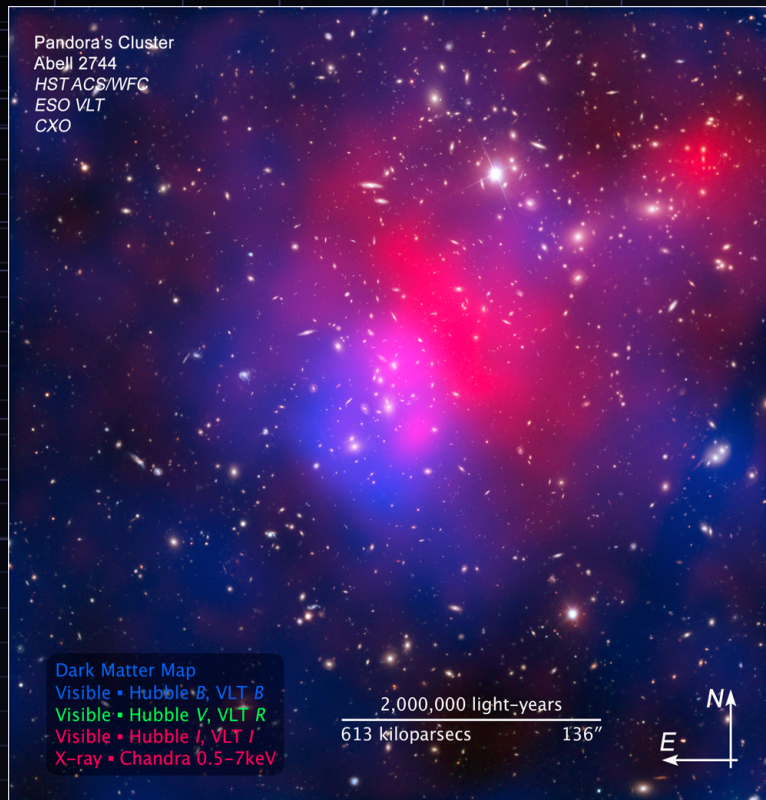
- Hierarchical structure formation
- Simplified baryons
- Laboratories for galaxy evolution

All clusters provide us the opportunity to directly or indirectly measure their *gravitational potentials*



- Lensing
- Dynamics
- Emission of the intra-cluster medium
- Scattering of the ICM

Use the most direct measures of the *gravitational potential* in clusters for frontier cosmology



- **Lensing**
- **Dynamics**
- Emission of the intra-cluster medium
- Scattering of the ICM

The Role of the Potential and Gravity



$$\Delta f = \nabla^2 f = \nabla \cdot \nabla f$$

$$\Delta f = \sum_{i=1}^n \frac{\partial^2 f}{\partial x_i^2}$$

The Laplacian $\Delta f(x,y,z)$ of a function at a position x,y,z is the rate at which the *average* of the function deviates from $f(x,y,z)$ as the distance increases. Zeros in the Laplacian define “edges”.

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \varphi(x, y, z) = f(x, y, z).$$



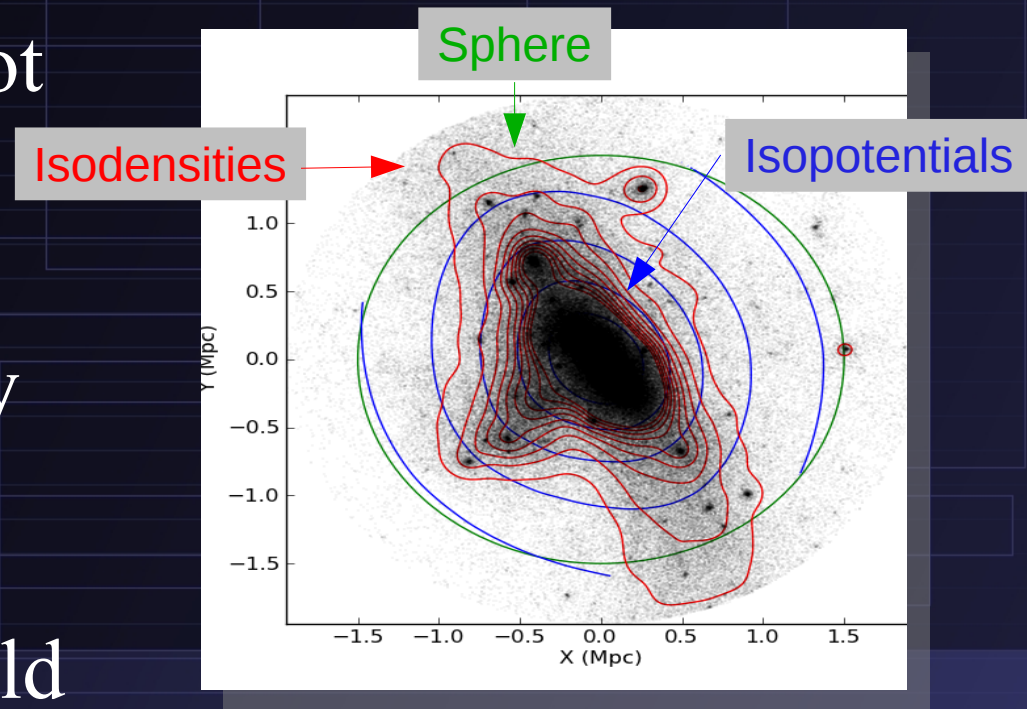
We apply Gauss's Law to a gravitational field and then the Laplacian to the resulting scalar potential field (LHS). The result is Poisson's Eq:

$$\nabla \cdot \mathbf{g} = -4\pi G\rho,$$

$$\nabla^2 \Phi = 4\pi G\rho.$$

Density and Potential in N-body Halos

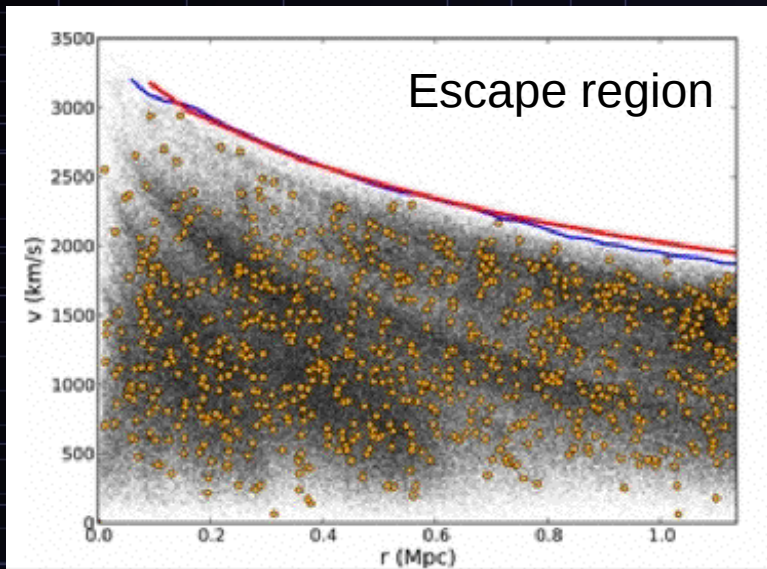
- Cluster-sized dark matter halos are not **spherical** in their **density**
- But they are nearly **spherical** in their local gravitational **potential** scalar field



From Escape Velocity to Mass: the radius-velocity phase space of clusters

$$\nabla^2\Phi = 4\pi G\rho.$$

$$v_{\text{esc}}^2(r) = -2\Phi(r).$$



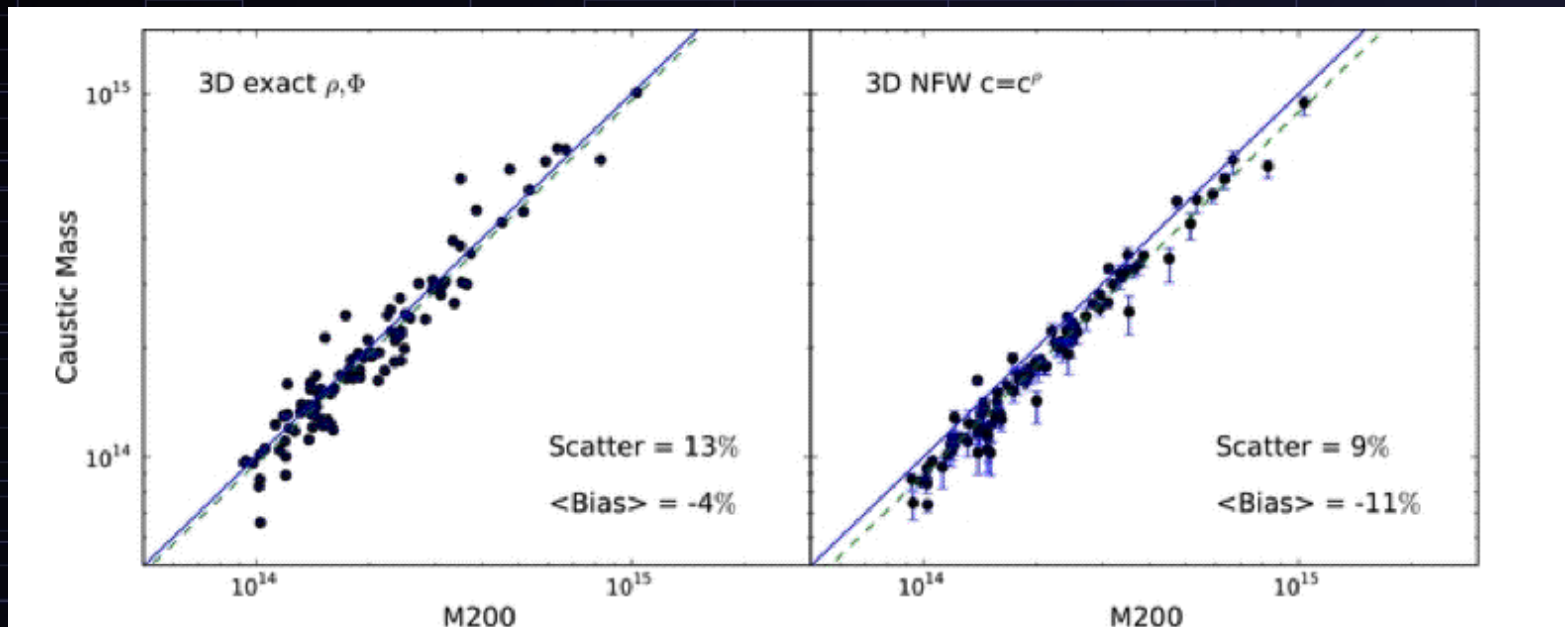
Gifford, Miller, Kern 2013



Wu et al.

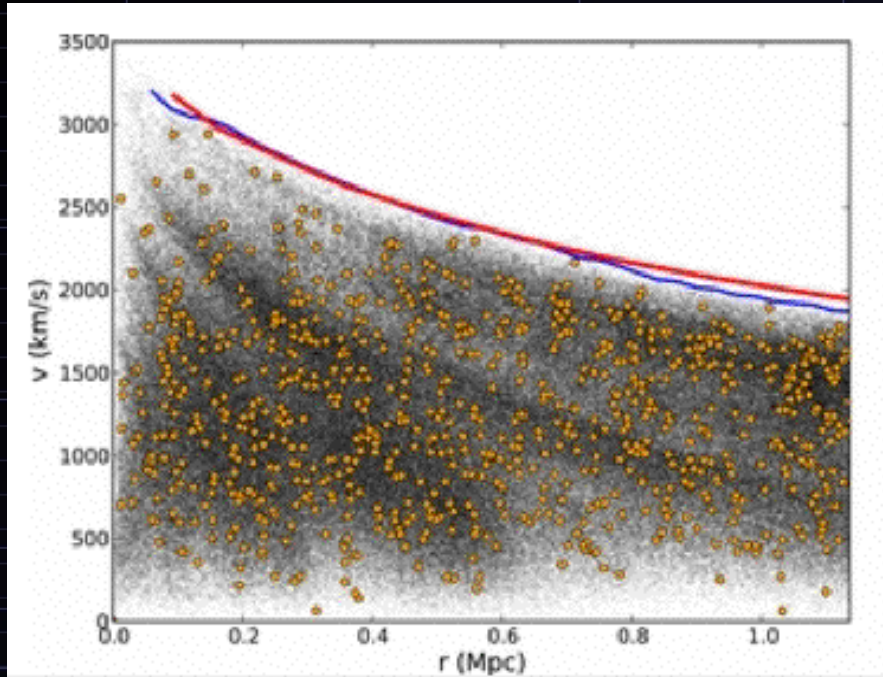
$$GM(<R) = \int_0^R \hat{\mathcal{F}}(r) v_{\text{esc}}^2(r) dr,$$

Escape Velocity Mass of N-body Halos

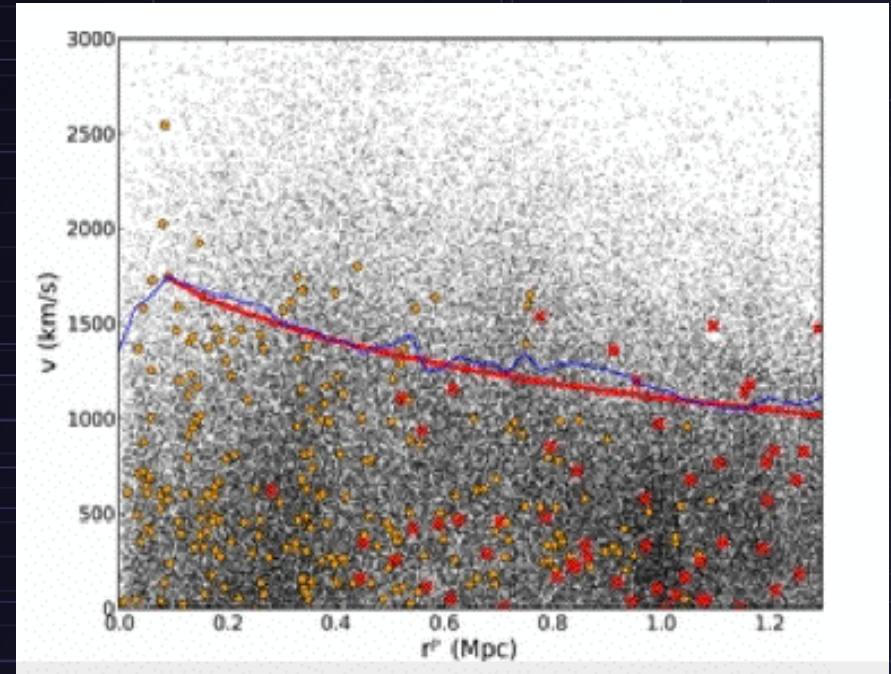


Gifford, Miller, Kern 2013

Projection and the Escape Velocity

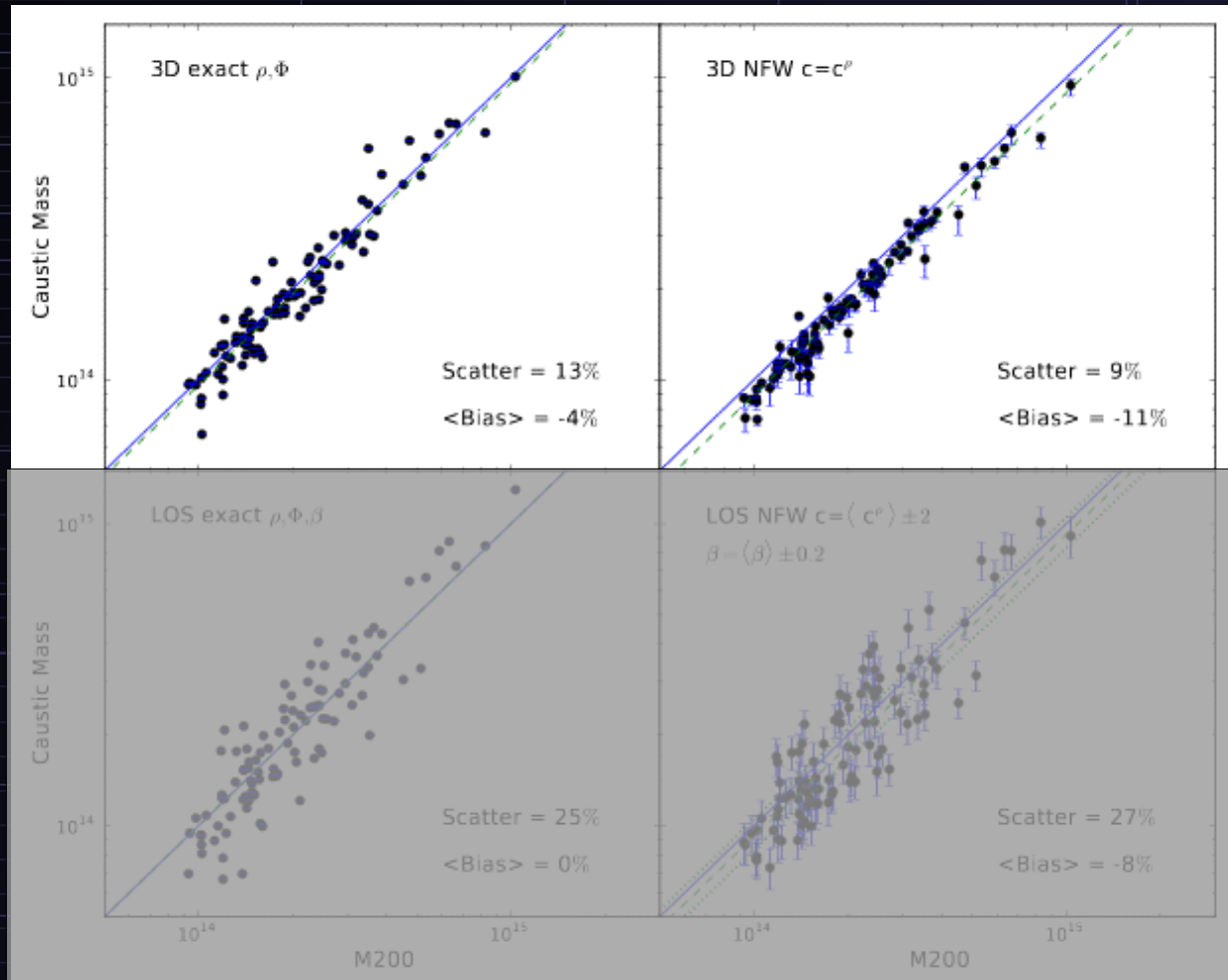


3D

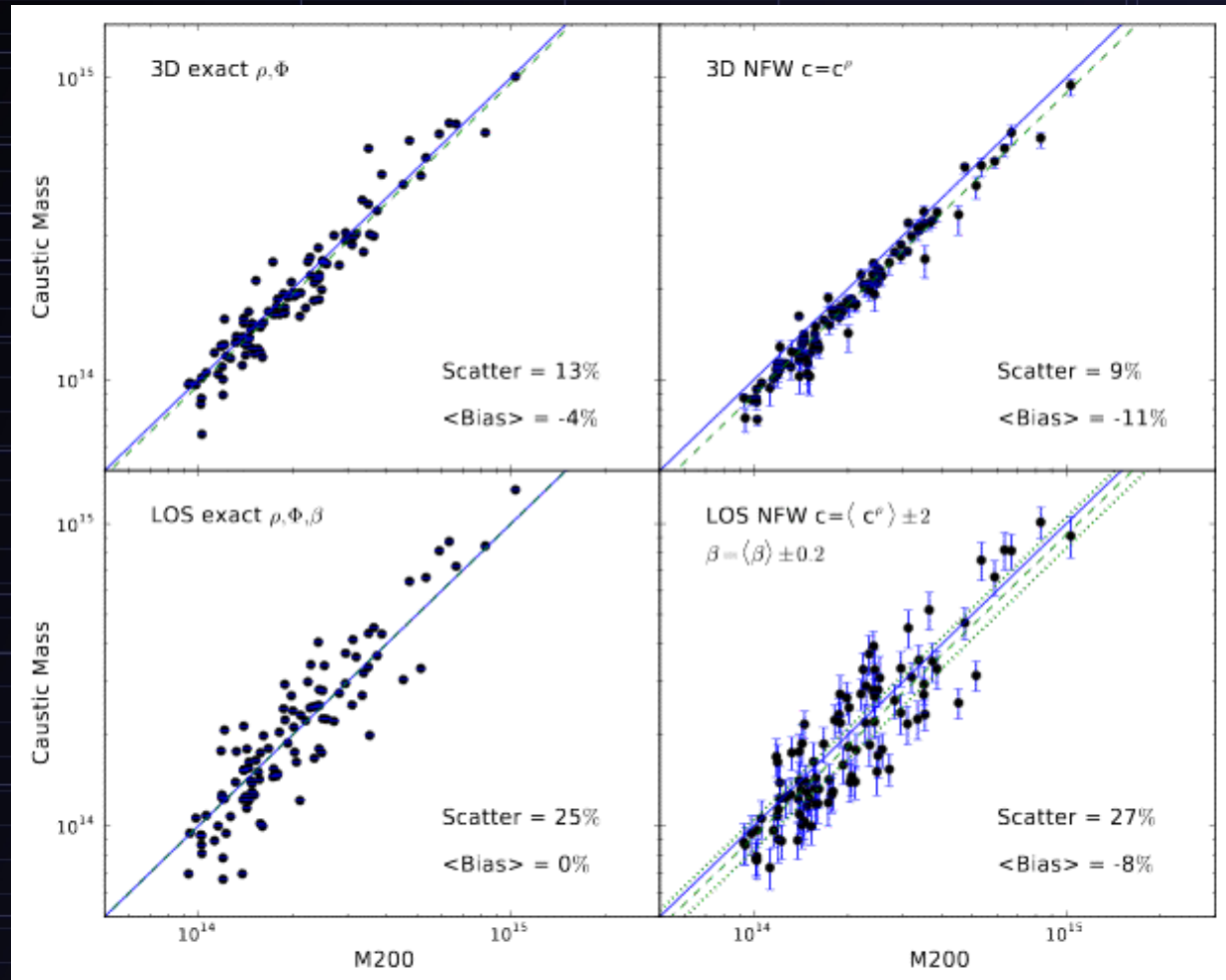


2D

Projected Escape Velocities

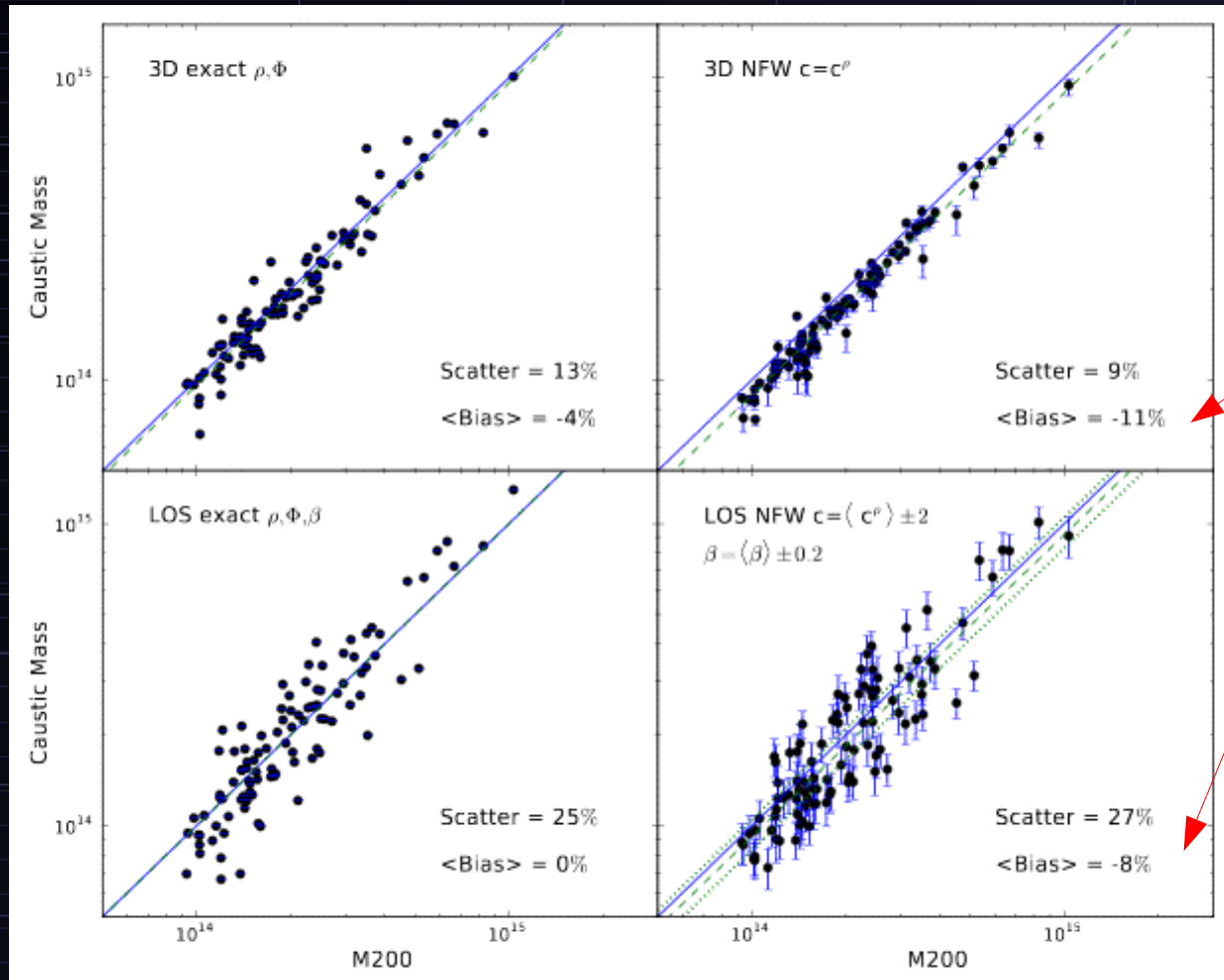


Projected Escape Velocities

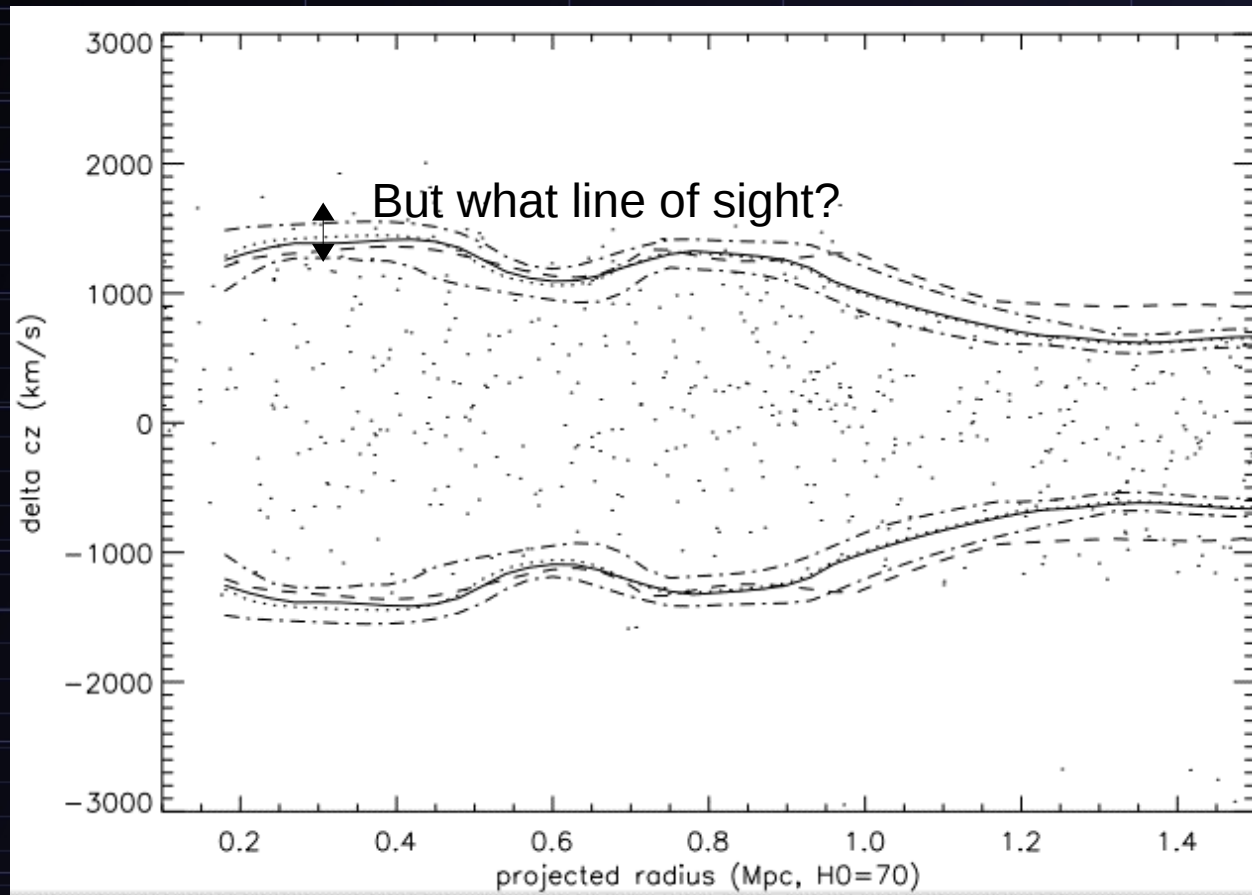


Projected Escape Velocities

Spherical NFW potential

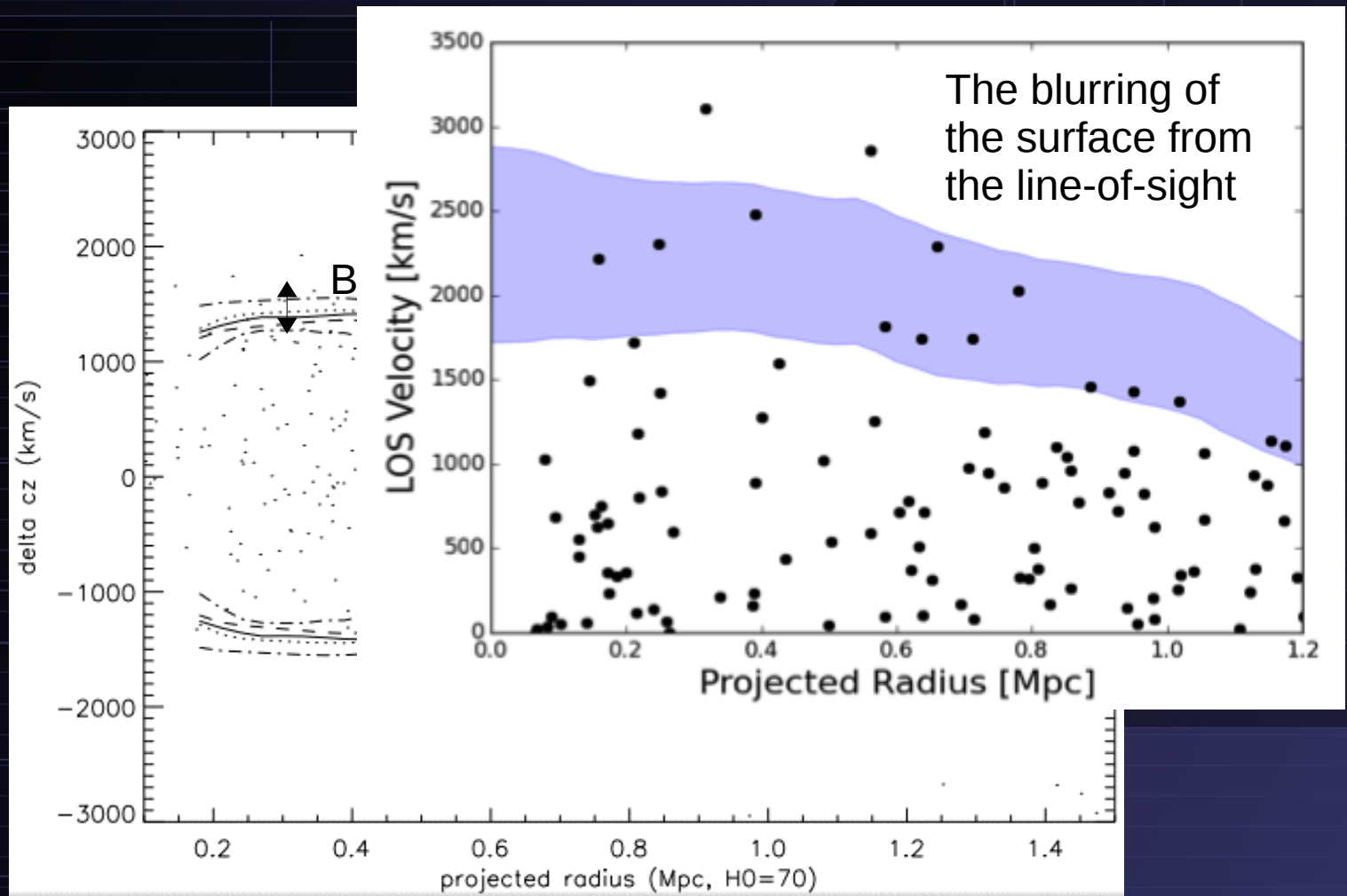


The Virgo Cluster: Real data

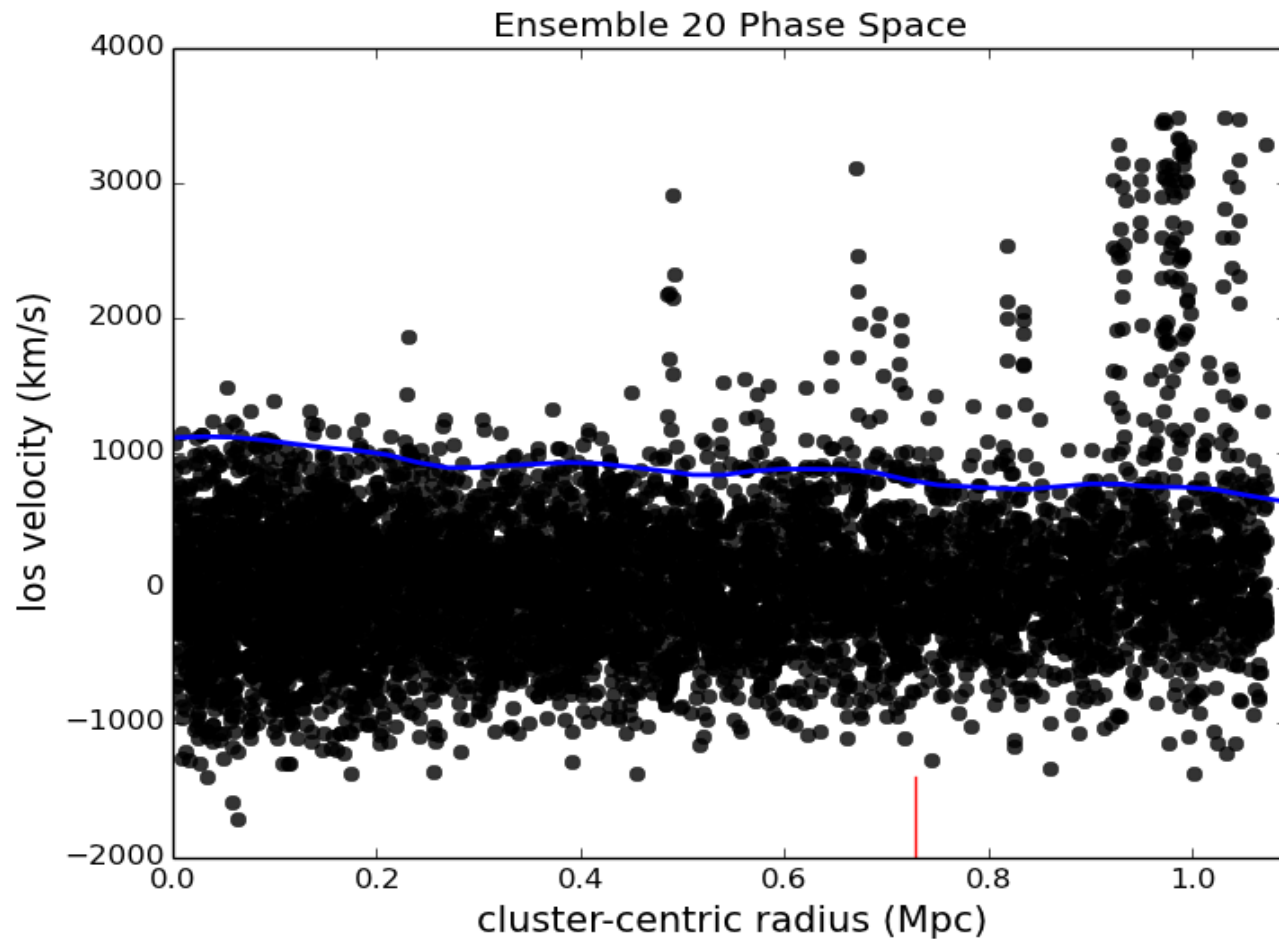


The Virgo Cluster: Real data

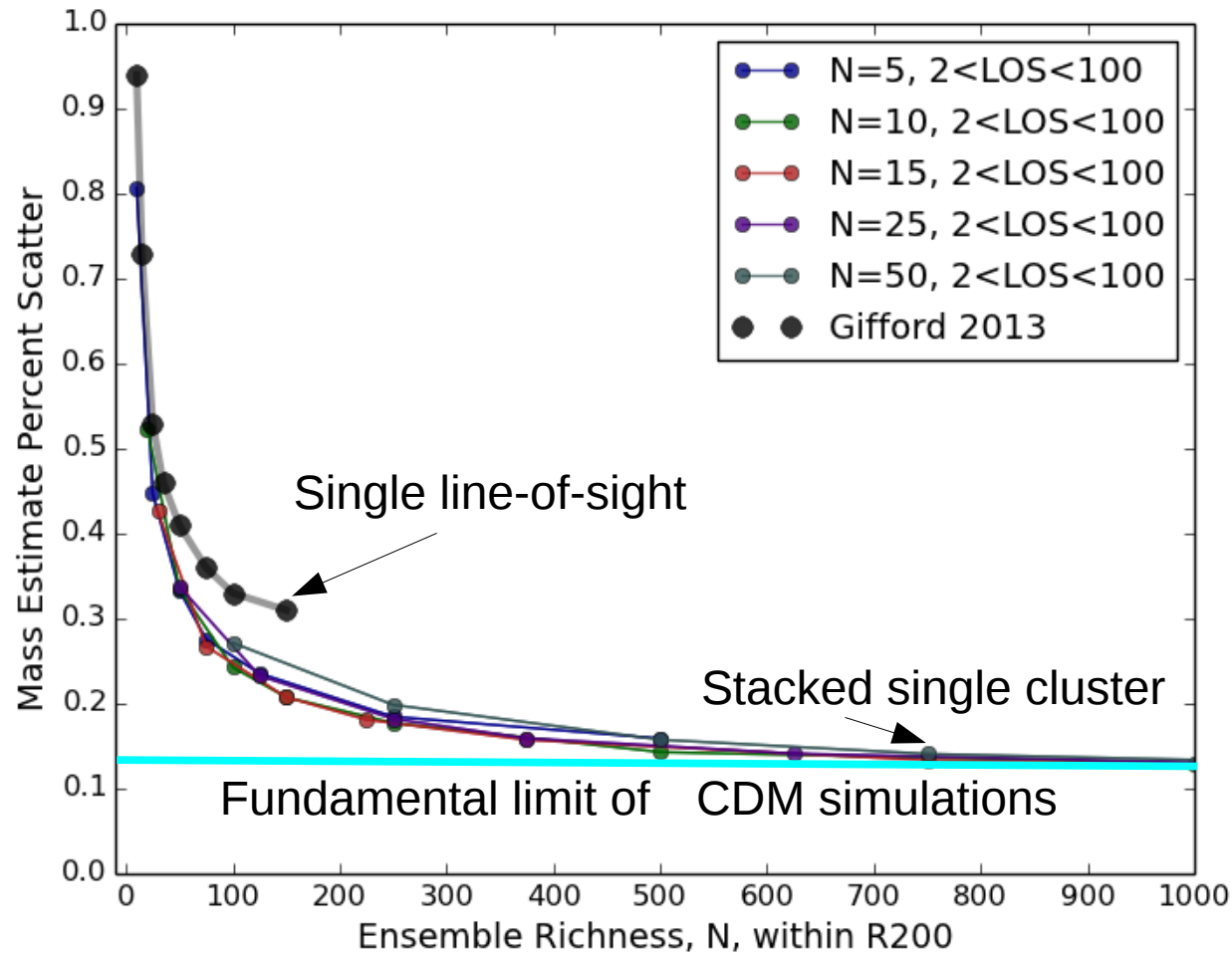
Simulations



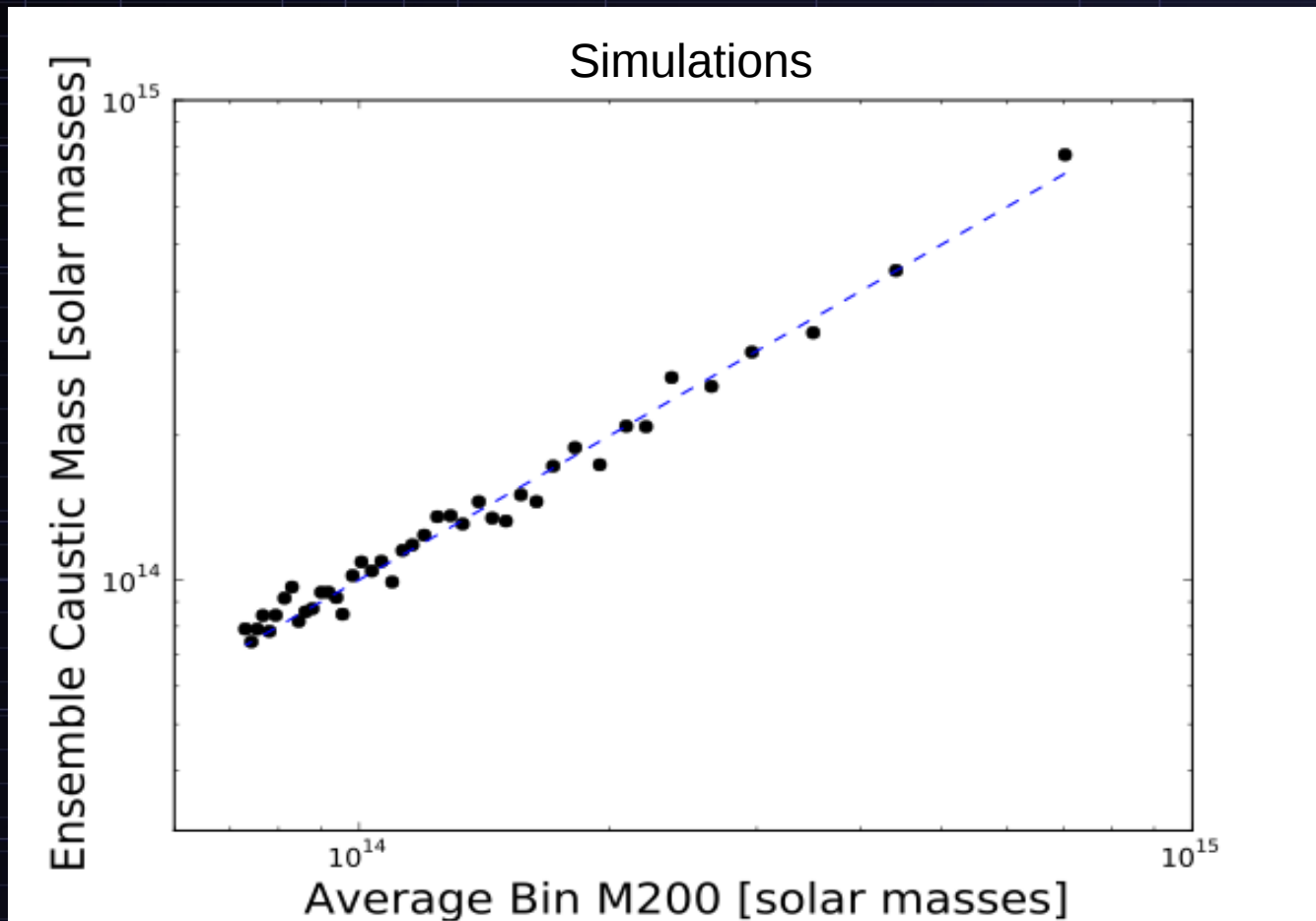
An ensemble phase space-2D



Solution to the line-of-sight issues: Stacking



Stacking based on a bin averaged
optical mass indicator (richness):
5% mass scatter, **0%** bias



Frontier Cosmology with Galaxy Clusters

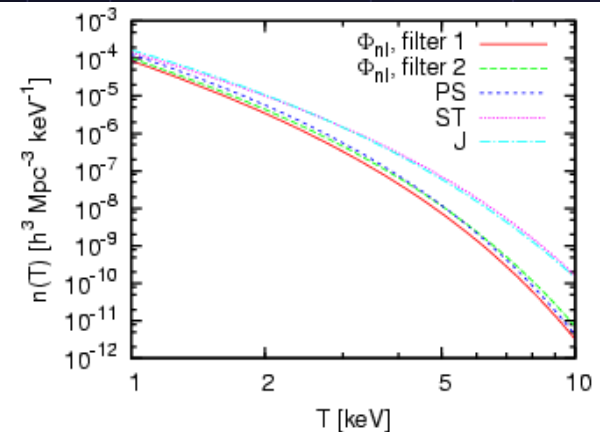
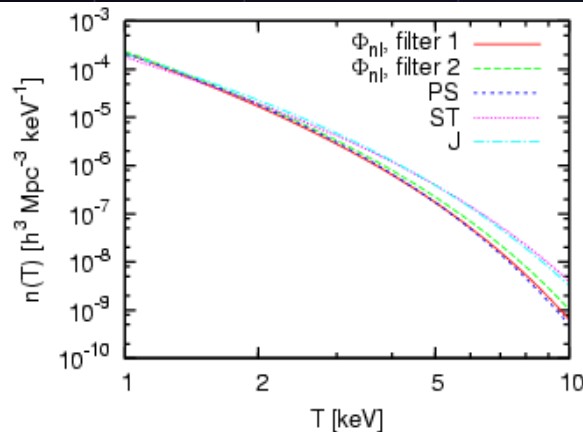
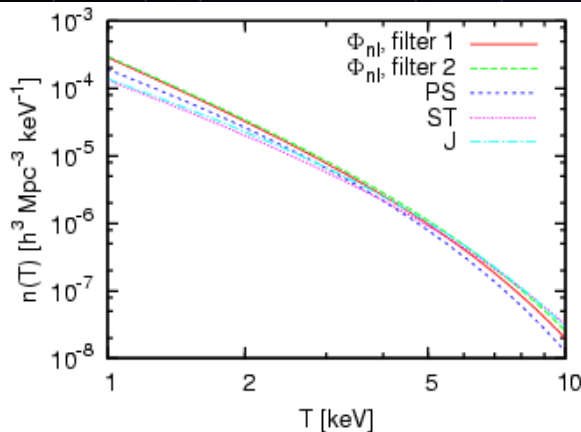
New directions

- ~~1. Direct mass estimates via their potentials~~
- 2. The “Potential Abundance” Function
- 3. Constraining
- 4. Testing gravity (alternatives to Dark Energy)

Part 2: Cosmology directly via the potential instead of the mass density

Angrick and Bartelmann 2009

$$\Delta\Phi_c(a) = \frac{3}{2} H_0^2 \Omega_{m0} \frac{\delta_c(a)}{a}, \quad n(\Phi) = \int_{\Delta\Phi_c}^{\infty} d(\Delta\Phi) \tilde{n}(\Phi, \Delta\Phi), \quad P_\Phi(k) = \frac{9}{4} \frac{\Omega_{m0}^2}{a^2} \frac{H_0^4}{k^4} P_\delta(k).$$



Linking the Potential from Photons and Dynamics

From the observed density (weak lensing)

$$\rho(r) = \frac{\rho_0}{(r/r_s)(1 + r/r_s)^2}$$

NFW

To the inferred mass profile

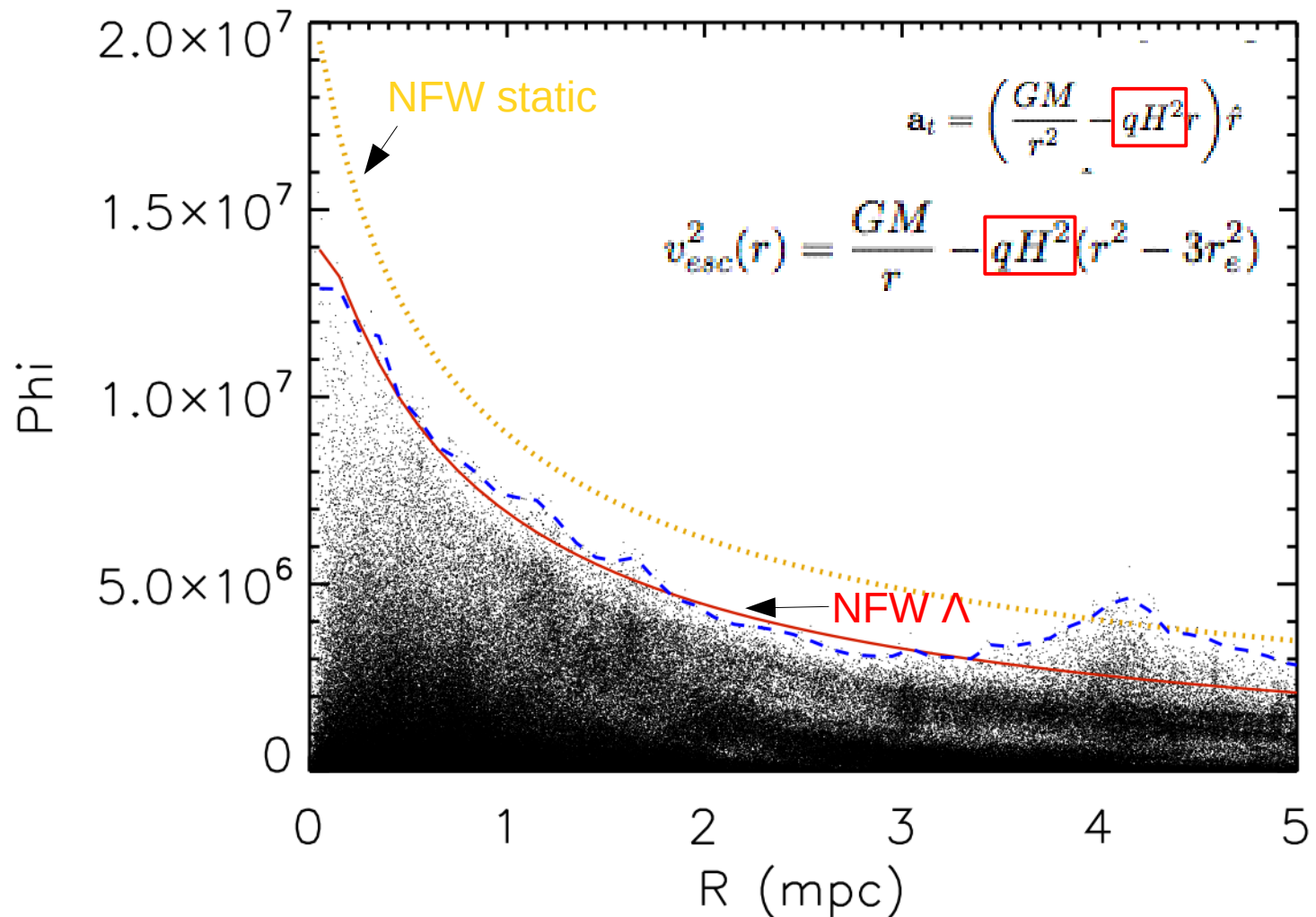
$$M(< r) = 4\pi\rho_0 r_s^3 \left[\ln \left(1 + \frac{r}{r_s} \right) - \frac{r/r_s}{1 + r/r_s} \right]$$

To the potential via the Poisson equation

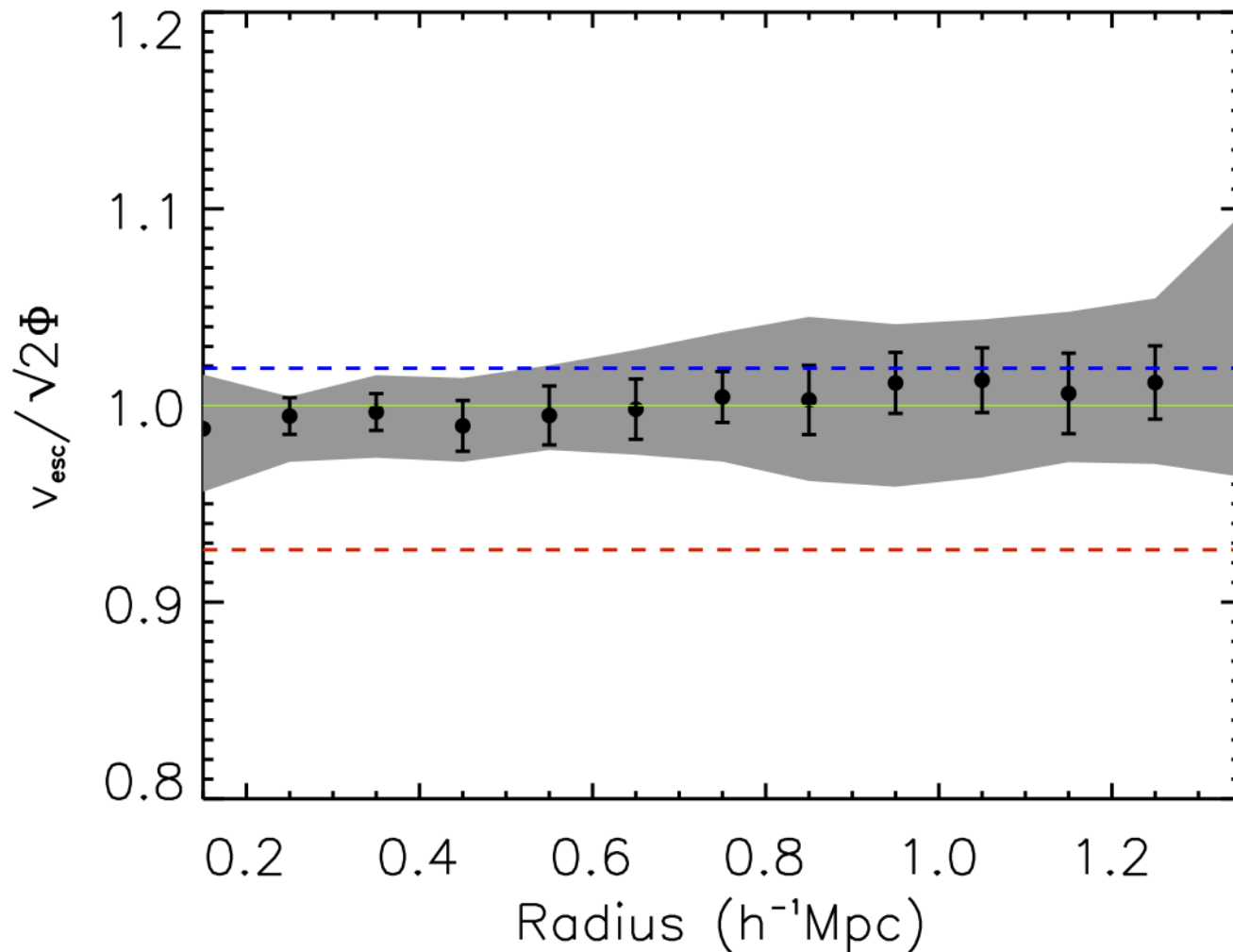
$$\Phi_{GR}(r) = \frac{-4\pi G \rho_s r_s^2 \ln(1 + r/r_s)}{r/r_s} + \Phi_0$$

$$\Phi(r) = -\frac{1}{2}v_{esc}^2(r)$$

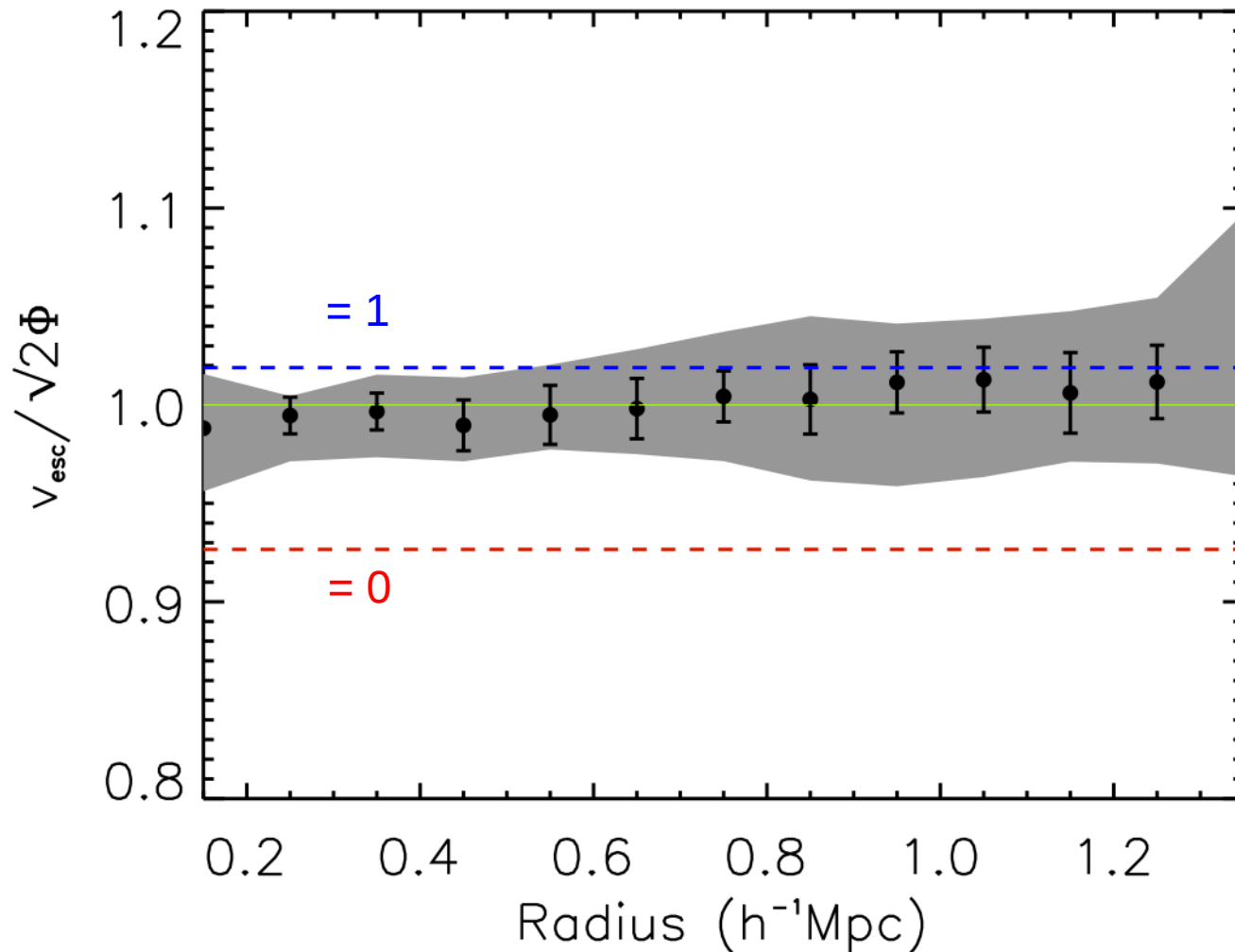
Enter: the expanding Universe



Part 3: Constraining the acceleration of the Universe



Part 3: Constraining the acceleration of the Universe



Testing gravity in low density regions and on Mpc Scales

modify the left hand side of Einstein Field Equations:

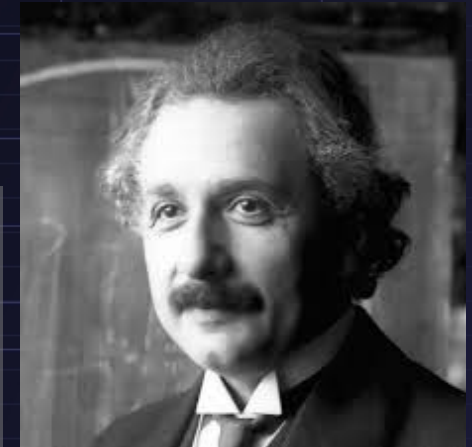
$$\nabla^2 \Phi = 4\pi G \rho$$

geometry = mass, energy

$$G_{\mu\nu} = 8\pi G (T_{\mu\nu}^M + T_{\mu\nu}^{DE})$$

formally equivalent to...

$$G_{\mu\nu} + \text{_____} = 8\pi G T_{\mu\nu}^M$$



$$G_{\mu\nu} + f_R R_{\mu\nu} - (1/2 f - \square f_R) g_{\mu\nu} - \nabla_\mu \nabla_\nu f_R = 8\pi G T_{\mu\nu}$$

modified gravitational potential in $f(R)$ gravity

Equation of motion for scalar field:

$$\nabla^2 \delta f_R = (1/3) [- (8\pi G) \delta \rho + \delta R(f_R)]$$

Modified Poisson equation:

$$\nabla^2 \Phi = (16\pi G/3) \delta \rho - (1/6) \delta R(f_R)$$

Combine...

modified gravitational potential:

$$\Phi = \phi_N - \frac{1}{2} \delta f_R$$

modified gravitational potential in $f(R)$ gravity

Equation of motion for scalar field ($f_R \equiv df(R)/dR$):

$$\nabla^2 \delta f_R = (1/3) [-(8\pi G) \delta \rho + \delta R(f_R)]$$

Perturbation from background value today (f_{R0}):

$$\delta f_R = f_R - f_{R0}$$

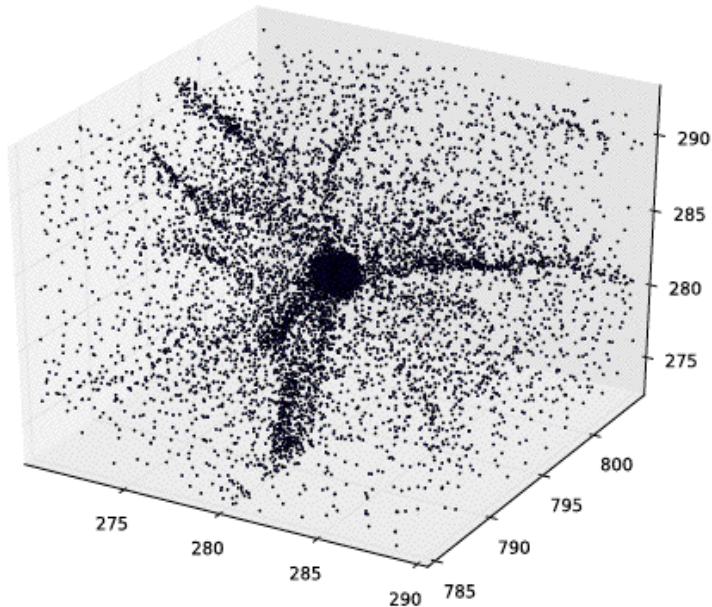
Quote modification “strength” as value of f_{R0} :

$$|f_{R0}| = 0 \equiv \text{LCDM (“GR”)}$$

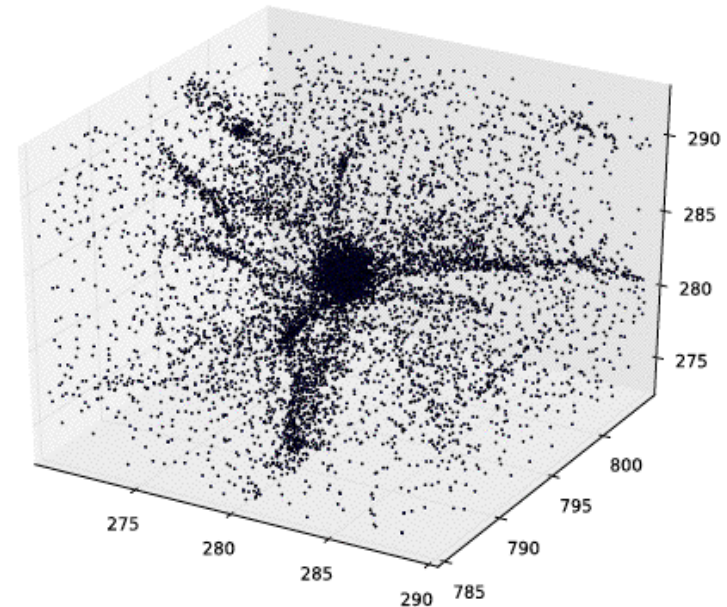
$$|f_{R0}| = 10^{-6} \equiv \text{FR6}$$

$$|f_{R0}| = 10^{-5} \equiv \text{FR5}$$

same galaxy cluster in two different universes...



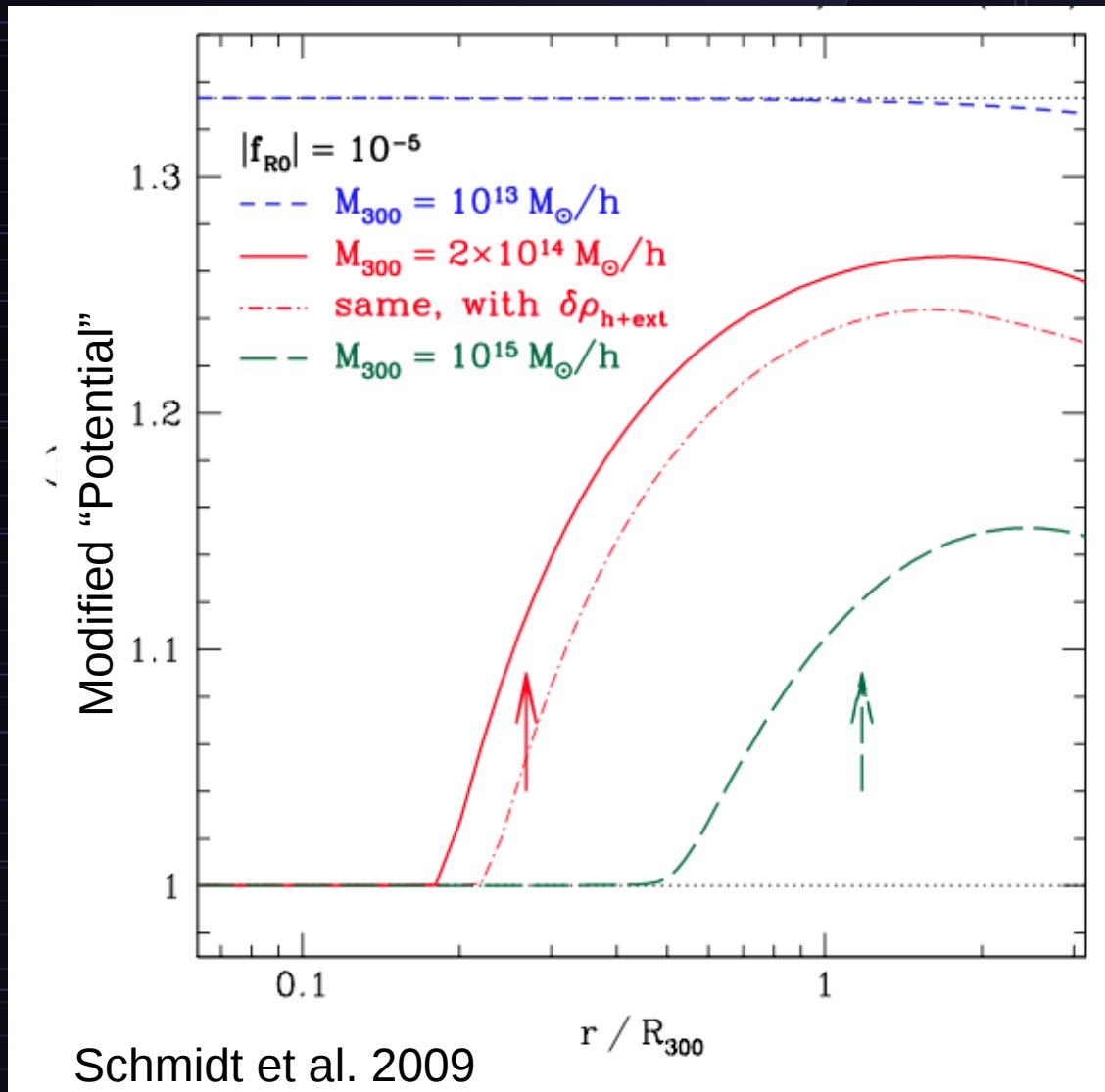
LCDM/GR



Chameleon $f(R)$ gravity

N-body simulations
by Kazuya Koyama, Gong-Bo Zhao, Baojiu Li

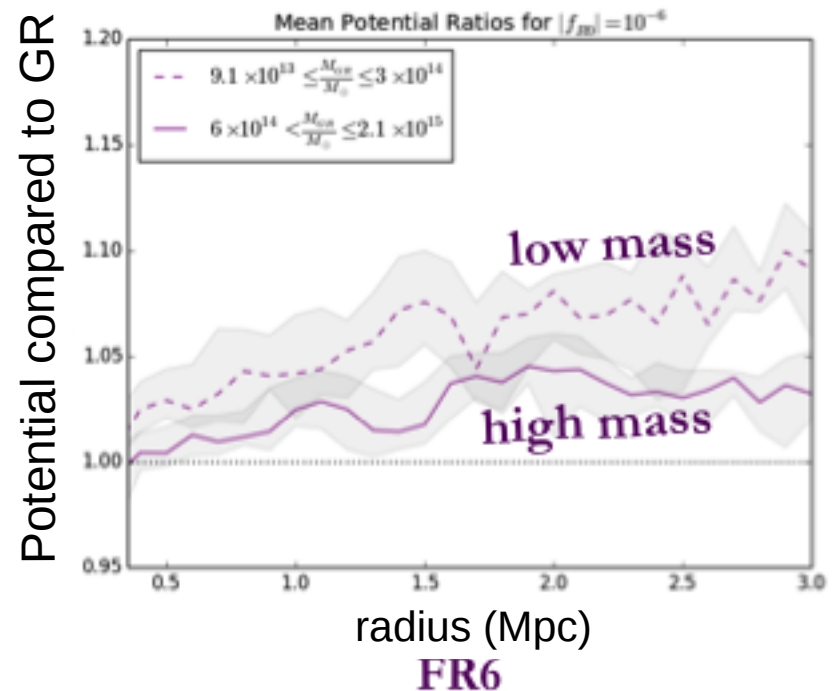
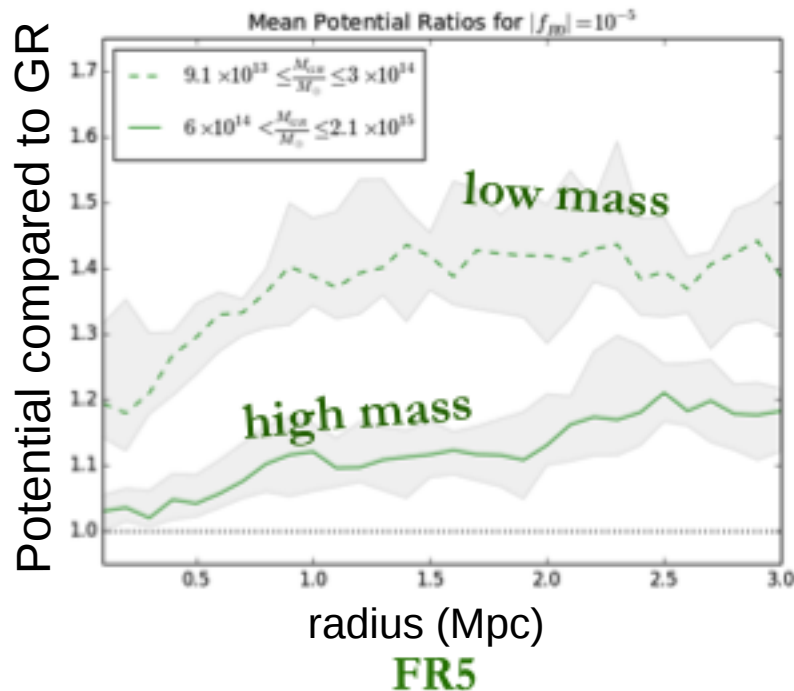
The Gravitational Potential Profile in $f(R)$ Gravity: Screening



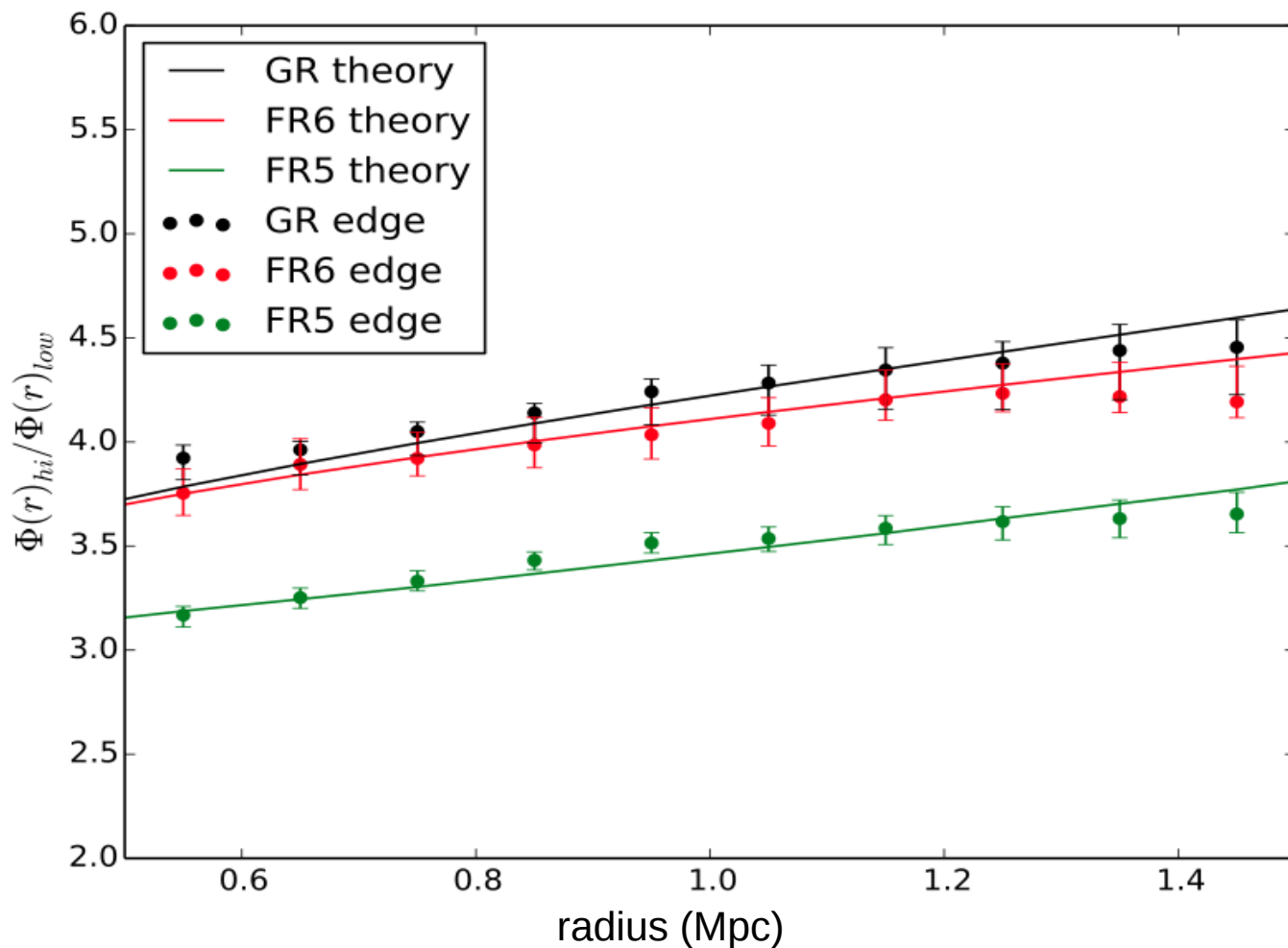
Clusters: scale and size

Stark, Miller, Koyama et al.

High mass clusters are *screened* vis-à-vis low mass clusters in $f(R)$ gravity, but not in GR



Part 4: Testing Alternatives to



Frontier Cosmology with Galaxy Clusters

New directions

- 1. Direct mass estimates via their potentials
- 2. The “Potential Abundance” Function
- 3. Constraining
- 4. Testing gravity (alternatives to Dark Energy)